1) the magnetic flux density on the Surface of an iron face is 1.8 T which is a typical Saturation Level value for ferromagnetic material, Find the force density on the iron face.

Sol

$B = 1.8\,T$

the Stored energy density

$$w_f = \frac{1}{2}\frac{B^2}{\mu}$$

$$W_f = w_f \times volume$$

A = Area of iron face in m^2

X = Distance b/w Surface

Let,

Volume = A X

$$W_f = \frac{1}{2} \times \frac{B^2}{\mu} \times A X$$

$$F_f = -\frac{\partial W_f}{\partial X} = \frac{-\partial}{\partial X}\left[\frac{1}{2}\frac{B^2}{\mu} A X\right]$$

$$\boxed{F_f = -\frac{1}{2}\frac{B^2 A}{\mu}}$$

$$\text{Force density} = \frac{F_f}{A} = \frac{-\frac{1}{2}\frac{B^2 \cancel{A}}{\mu}}{\cancel{A}} = -\frac{1}{2}\frac{B^2}{\mu}$$

$$= \frac{1}{2}\frac{B^2}{\mu}$$

$$= \frac{1}{2}\frac{B^2}{\mu}$$

$$= \frac{1}{2} \times \frac{(1.8)^2}{4\pi \times 10^{-7}}$$

$$\boxed{\text{magnitude of force density} = 1.2891 \times 10^{6}\ N/m^2}$$

2) The $\lambda - i$ characteristics of singly excited electromagnet is given by $i = 121\lambda^2 x^2$ for $0 < i < 4A$ and $0 < x < 10$ cm. If the air gap is 5 cm and a current of 3A is flowing in the coil, calculate i) Field energy ii) Co-energy iii) Mechanical force on the moving part.

given

$$i = 121\lambda^2 x^2$$

$$x = 5\ cm = 5 \times 10^{-3}\ m$$

$$i = 3A$$

i) Field energy.

$$W_f = \int_0^{\lambda} i\, \lambda\, d\lambda$$

$$\cancel{W} = \int_0^{\lambda} 121\,\lambda^2 x^2\, d\lambda$$

$$= 121x^2\left[\frac{\lambda^3}{3}\right]_0^{\lambda}$$

$$\boxed{W_f = 121\,\frac{\lambda^3}{3}\,x^2}$$

Let

$$\lambda^2 = \frac{i}{121x^2} = \frac{3}{121\times(5\times10^{-2})^2} = 9.917 \quad \boxed{\lambda^2 = \sqrt{9.97}}$$

$$\lambda = 3.1491$$

$$W_f = 121\times\frac{(3.1491)^3}{3}\times(5\times10^{-2})^2 = 3.1491$$

$$W_f = 3.1491\,J$$

ii) Co-energy.

$$W_f' = \int_0^{i} \lambda\, di \qquad \lambda = \sqrt{\frac{i}{121x^2}}$$

$$= \frac{(i)^{1/2}}{11x}$$

$$W_f' = \int_0^{i} \frac{(i)^{1/2}\,di}{11x} = \frac{1\,(i)^{3/2}}{11\times 3/2} = \frac{2}{33x}(i)^{3/2}$$

$$W_f' = \frac{2}{33}\times\frac{1}{5\times10^{-2}}\times(3)^{3/2}$$

$$\boxed{W_f' = 6.2983\,J}$$

(iii) $$F_f = -\frac{\partial w_f(\lambda, x)}{\partial x}$$

$$= -\frac{\partial}{\partial x}\left[\frac{121}{3}\lambda^3 x^2\right]$$

$$= -\frac{121}{3} \times \lambda^3 \times 2x$$

$$= -\frac{121}{3} \times (3.1491)^3 \times 2 \times (5 \times 10^{-2})$$

$$\boxed{F_f = -125.95\ N}$$

3) In a rectangular electromagnetic relay, the exciting coil has 1500 turns of resistance 1Ω, the Cross - Sectional area of the Core $A = 5$ cm $\times 5$ cm. Neglect the reluctance of magnetic circuit & fringing effects. If the coil is excited with an a.c. voltages of 50Hz frequency, having peak to peak value of 100 V and the armature is held at a fixed distance of 1cm, find the average force on the armature

the reluctance of magnetic circuit is to be neglected.

$$S_g - \text{Reluctance of air gap} = \frac{l_g}{\mu_0 A}$$

$$l_g = 1\text{cm} = 1 \times 10^{-2}\ m$$

$$A = 25\ \text{cm}^2 = 25 \times 10^{-4}$$

(5)

$$S_g = \frac{1\times10^{-2}}{4\pi\times10^{-7}\times25\times10^{-4}}$$

$$= 3.18309\times10^{6}\ AT/wb$$

$$L = \frac{N^2}{S_g} = \frac{(1500)^2}{3.18309\times10^{6}}$$

$$L = 0.7068\ H$$

$$X_L = 2\pi fL = 0.7068\times2\pi\times50$$

$$X_L = 222.066\ \Omega$$

$$Z_{coil} = r + jX_L$$

$$= 1 + j\,222.066 = 222.068\angle 89.74^\circ\ \Omega$$

Peak to peak value $V_{p.p} = 100V$,

$$V_m = \frac{V_{p.p}}{2} = 50V,$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 35.355\ V$$

$$I_{rms} = \frac{V_{rms}}{Z_{coil}} = \frac{35.355}{222.068} = 0.1592\ A$$

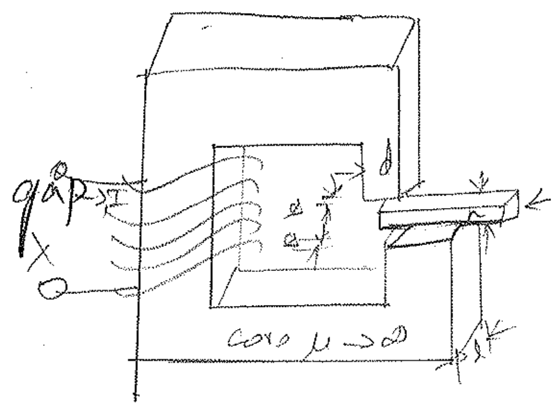
$$\phi = \frac{N I_{rms}}{S_g} = \frac{1500\times0.1592}{3.18306\times10^{6}} = 7.5026\times10^{-5}\ Wb$$

$$\boxed{B = \frac{\phi}{A} = 0.03\ Wb/m^3}$$

$$F = \frac{1}{2}\frac{B^2A}{\mu_0} = \frac{1}{2}\times\frac{(0.03)^2\times(25\times10^{-4})}{4\pi\times10^{-7}} = 0.8952\ N$$

4) The relay shown in . is made from infinitely (6)
permeable magnetic material with a movable plunger also of infinitely permeable material. The height of the plunger is much greater than the air gap length ($h \gg g$). Calculate the magnetic energy stored as a function of plunger position ($0 < x < d$) for $N = 1000$ turns, $g = 2.0$ mm, $d = 0.5$ m, $l = 0.1$ m and $I = 10$ A.

Sol:-



Cross Sectional area of air gap

$$a_g = l(d - x)$$

$$= ld\left(1 - \frac{x}{d}\right)$$

μ magnetic path is ∞

reluctance is zero

$$\therefore L = \frac{N^2}{\mathcal{R}}$$

$$L(x) = \frac{N^2 \mu_0 a_g}{2g} = \frac{N^2 \mu_0 ld\left(l - \frac{x}{d}\right)}{2g}$$

$$\mathcal{R} = \frac{l_g}{\mu_0 a_g} \quad \& \quad l_g = 2g$$

$$= -4.1667\left[\frac{-1}{x}\right]_{0.5}^{1}$$

$$= -4.1667\left[\frac{-1}{0.5} - \frac{1}{1}\right]$$

$$= -4.1667[1]$$

$$\boxed{\Delta W_m = -4.1667\ J}$$

(ii) ΔW_{e1} = Energy Supplied by Source 1.

$$= \int_{\lambda_1 \text{ at } x_1}^{\lambda_1 \text{ at } x_2} i_1\, d\lambda_1$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$= \left(3 + \frac{1}{3x}\right) 10 + \left(\frac{1}{3x}\right)(-5)$$

$$= 30 + \frac{10}{3x} + \left(\frac{-5}{3x}\right)$$

$$= 30 + \frac{10}{3x} - \frac{5}{3x} \quad = 30 + \frac{5}{3x} \quad = 30 + \frac{1.667}{x}$$

put λ_1 at $x_1 = 0.5$ $\qquad\qquad$ λ_1 at $x_2 = 1$

$$= 30 + \frac{1.667}{0.5} = 33.33 \qquad\qquad = 30 + \frac{1.667}{1} = 31.667$$

$$\Delta W_{e1} = i_1 \int_{\lambda_1 \text{ at } x_1}^{\lambda_1 \text{ at } x_2} d\lambda_1$$

Sol

Coil are excited by Constant Current.

Co-energy expressions to be used.

$$W_f'(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \frac{1}{2}\left[3+\frac{1}{3x}\right](10)^2 + \left(\frac{1}{3x}\right)(10)(-5) + \frac{1}{2}\left[1+\frac{1}{3x}\right](-5)^2$$

$$= 150 + \cancel{\frac{50}{3x}} - \cancel{\frac{50}{3x}} + 12.5 + \frac{12.5}{3x}$$

$$W_f' = 162.5 + \frac{4.1667}{x}$$

(i)

$$F_f = \frac{\partial W_f'}{\partial x}$$

$$= \frac{\partial}{\partial x}\left[162.5 + \frac{4.1667}{x}\right] \qquad \boxed{\left[\frac{1}{x}\right]\frac{\partial}{\partial x} = -\frac{1}{x^2}}$$

$$\boxed{F_f = -\frac{4.1667}{x^2}}$$

$\Delta W_m \Rightarrow$ mechanical work done

$(0.5 \to 1)$

$$= \int_{0.5}^{1} F_f \, dx$$

$$= \int_{0.5}^{1} \frac{-4.1667}{x^2} \, dx$$

$$W_{field} = \frac{1}{2}\frac{\psi^2}{L(x)} \quad , \quad \psi = L(x)\, i ,$$

$$W_{field} = \frac{1}{2}\frac{L^2(x)\, i^2}{L(x)}$$

$$= \frac{1}{2} L(x)\, i^2$$

$$W_{field} = \frac{1}{2} \times \frac{N^2 \mu_0 l d \left(1 - \frac{x}{d}\right)}{2g} i^2 \; J$$

$$W_{field} = \frac{1}{2} \times \frac{(1000)^2 \times 4\pi \times 10^{-7} \times 0.1 \times 0.5 \times \left(1 - \frac{x}{0.5}\right)}{2 \times 2 \times 10^{-3}}$$

$$W_{field} = 7.857\,(1 - 2x) \; J.$$

5) In an electromagnetic relay, functional relation b/w the current i in the excitation coil, the position of armature is x and the flux linkage ψ is given by $i = 2\psi^3 + 3\psi(1 - x + x^2)$ $x > 0.5$. Find force on the armature as a function of ψ.

Sol :-

$$i = 2\psi^3 + 3\psi\,(1 - x + x^2)$$

$$i = 2\psi^3 + 3\psi - 3\psi x + 3\psi x^2$$

$$W_f = \int_0^{\psi} i(\psi)\, d\psi$$

$$= \int_0^{\psi} [2\psi^3 - 3\psi - 3\psi x + 3\psi x^2]\, d\psi$$

$$= \left[\frac{\overset{1}{\cancel{2}}\psi^4}{\cancel{4}_2} - \frac{3\psi^2}{2} - 3x\frac{\psi^2}{2} + 3x^2\frac{\psi^2}{2} \right]_0^{\psi}$$

$$= \frac{\psi^4}{2} + \frac{\psi^2}{2}[3 - 3x + 3x^2]$$

$$= \frac{\psi^4}{2} + \frac{3\psi^2}{2}[x^2 - x - 1]$$

$$F_f = \frac{-dW_f(\psi, x)}{dx}$$

$$= -\frac{d}{dx}\left[\frac{\psi^4}{2} + \frac{3\psi^2}{2}[x^2 - x - 1] \right]$$

$$\boxed{F_f = -\frac{3\psi^2}{2}(2x - 1)\ N}$$

6. Two Coupled Coils have Self & mutual inductance of $L_{11} = 3 + \frac{1}{3x}$; $L_{22} = 1 + \frac{1}{3x}$; $L_{12} = L_{21} = \frac{1}{3x}$ over a Certain range of linear displacement x. The first Coil is excited by a Constant Current of 10 A and the Second by a Constant Current of -5 A. Find the mechanical work done if x changes from 0.5 to 1 m and energy Supplied by each electrical Source for the above Case.

$$= i_1[\lambda_1 \text{ at } x_2 - \lambda_1 \text{ at } x_1]$$

$$= 10[31.667 - 33.33]$$

$$\Delta W_{e1} = -16.667 \text{ J}$$

Similarly

ΔW_{e2} = Energy Supplied by Source 2.

$$\Delta W_{e2} = \int_{\lambda_2 \text{ at } x_1}^{\lambda_2 \text{ at } x_2} i_2 \, d\lambda_2$$

$$= i_2[\lambda_2 \text{ at } x_2 - \lambda_2 \text{ at } x_1]$$

$$\lambda_2 = L_{12} i_1 + L_{22} i_2$$

$$= \left(\frac{1}{3x}\right) 10 + \left(1 + \frac{1}{3x}\right)(-5)$$

$$= \frac{10}{3x} - 5 - \frac{5}{3x}$$

$$= -5 + \frac{1.667}{x}$$

$$\lambda_2 \text{ at } x_1 = 0.5, \quad = -5 + \frac{1.667}{0.5} = -1.667$$

$$\lambda_2 \text{ at } x_2 = 1, \quad = -5 + 1.667 = -3.333$$

$$= \int i_2 \, d\lambda_2 = i_2[x_2 - x_1]$$

$$\Delta W_{e2} = -5[-3.333 - (-1.667)]$$

$$\Delta W_{e2} = 8.33 \text{ J}$$

Net electrical i/p $= \Delta W_{e1} + \Delta W_{e2}$

$$= -16.667 + 8.33 = -8.33 \text{ J}$$

(7) For doubly excited magnetic field system, various inductances are, $L_{11} = (4 + \cos 2\theta) \times 10^{-3}$ H, $L_{12} = 0.15 \cos\theta$ H, $L_{22} = (20 + 5\cos 2\theta)$ H Find the torque developed if $i_1 = 1A$ & $i_2 = 0.02A$.

Sol

$$W_f'(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \frac{1}{2}(4 + \cos 2\theta) \times 10^{-3} + 0.15\cos\theta \times 0.02 + \frac{1}{2}(20 + 5\cos 2\theta) \times (0.02)^2$$

$i_1 = 1A$, $i_2 = 0.02A$

$$= 6 \times 10^{-3} + 1.5 \times 10^{-3} \cos 2\theta + 3 \times 10^{-3} \cos\theta$$

$$T_f = \frac{\partial W_f'}{\partial \theta}$$

$$= \frac{\partial}{\partial \theta}\left(6 \times 10^{-3} + 1.5 \times 10^{-3} \cos 2\theta + 3 \times 10^{-3} \cos\theta\right)$$

$$\boxed{T_f = -3 \times 10^{-3} \sin 2\theta - 3 \times 10^{-3} \sin\theta}$$

(8) Two windings, one mounted in stator & other at rotor have self & mutual inductance of $L_{11} = 4.5$ and $L_{22} = 2.5$. $L_{12} = 2.8 \cos\theta$ H, where θ is the angle between axes of winding. Winding 2 is short circuited and current in winding as a function of time is $i_1 = 10 \sin\omega t$ A.

i) Determine the expression for numerical value in newton - meter for the instantaneous value of torque in terms of θ.

ii) Compute the time average torque in newton - meter when $\theta = 45^\circ$.

iii) If the rotor is allowed to move, will it continuously rotate or it will come to rest? if later at which value of θ_0

Sol :-

$$L_{11} = 4.5\,H, \quad L_{22} = 2.5\,H, \quad L_{12} = 2.8\cos\theta\,H$$

$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

$$= \frac{\partial}{\partial \theta}\frac{1}{2}L_{11} i_1^2 + \frac{\partial L_{12}}{\partial \theta} i_1 i_2 + \frac{1}{2}\frac{\partial L_{22}}{\partial \theta} i_2^2$$

$$= 0 - 2.8 \sin\theta\, i_1 i_2 + 0$$

$$= -2.8 \sin\theta\, i_1 i_2$$

$$V_m \cos\omega t = 4.5\frac{di_1}{dt} + [2.8\cos\theta]\frac{di_2}{dt}$$

$$0 = [2.8\cos\theta]\frac{di_1}{dt} + 2.5\frac{di_2}{dt}$$

$$\frac{di_2}{dt} = -\frac{2.8}{2.5}\cos\theta\frac{di_1}{dt}$$

ie $i_2 = -1.12\cos\theta\, i_1$

$i_1 = 10 \sin \omega t$

$i_2 = -1.12 \times 10 \cos\theta \sin \omega t$

i) $T_t = -2.8 \sin\theta \times [-11.2 \cos\theta \sin\omega t] \times 10 \sin\omega t$

$= 313.6 \sin\theta \cos\theta \sin^2\omega t$

ii) average torque,

$\frac{1}{T}\int_0^T \sin^2\omega t = \frac{1}{2}$ hence

$$T_{av} = \frac{313.6 \sin\theta \cos\theta}{2}$$

$$= 78.4 \sin 2\theta$$

$\theta = 45^\circ$,

$T_{av} = 78.4 \sin 2 \times 45^\circ$

$$\boxed{T_{av} = 78.4 \text{ Nm}}$$

(iii)

$\theta_0 = \pi/2$,

ie, 90°

9) A 6 pole, wave Connected d.c. armature has 300 Conductors & runs at 1200 r.p.m. If the useful flux per pole is 0.033 Wb. find the generated e.m.f

Sol:-

$P = 6$, $Z = 300$, $\phi = 0.033\ Wb$

$N = 1200$ rpm

wave winding $A = 2$

$$E_g = \frac{\phi P N Z}{60 A} = \frac{0.033 \times 6 \times 1200 \times 300}{60 \times 2} = 594\ V$$

$$\boxed{E_g = 594 V}$$

(10) A 4 pole, dc machine has a lap Connected armature having 60 Slots with 8 Conductors per Slot. The flux per pole is 30 mWb. If the armature is rotated at 1000 rpm. find the emf available across the armature terminals.

Sol:

$P = 4$, Lap winding $A = P = 4$ Slot $= 60$

$8 =$ Conductor / Slot

$\phi = 30 \times 10^{-3}\ Wb$, $N = 1000$ rpm

$Z =$ Slots $\times$ Conductors / Slot

$Z = 60 \times 8 = 480$

$$E_g = \frac{\phi P N Z}{60 A} = \frac{30 \times 10^{-3} \times 4 \times 1000 \times 480}{60 \times 4}$$

$$\boxed{E_g = 240\,V}$$

(11). An armature of a three phase alternator has 120 slots. The alternator has 8 poles. Calculate its distribution factor

<u>Sol</u>:

$$n = \frac{\text{Slots}}{\text{pole}} = \frac{120}{8} = 15$$

$$m = \text{Slots/pole/phase} = \frac{n}{3} = 15/3 = 5$$

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{15} = 12^\circ$$

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{5 \times 12}{2}\right)}{5 \times \sin\left(12/2\right)} = 0.957$$

$$\boxed{K_d = 0.957}$$

(12) In a 4 pole, 3 phase alternator, armature has 36 Slots. it is using an armature winding which is Short pitched by one slot. Calculate its coil span factor.

<u>Sol</u>:-

$$n = \frac{\text{Slots}}{\text{pole}} = \frac{36}{4} = 9$$

$$\beta = \frac{180^\circ}{9} = 20^\circ$$

α = Angle of Short pitch = 20°

$$k_c = \cos\left(\frac{\alpha}{2}\right) = \cos\left(\frac{20}{2}\right)$$

$$= \cos(10)$$

$$\boxed{k_c = 0.9848}$$

(13) Determine the breadth and pitch factors for a 4-pole, 3 phase winding with 2 slots/pole/phase coil span is 5 slot pitches.

Sol.

$P = 4$, $m = 2$ Coil Span = 5 Slot pitches.

n = m x number of phases

$= 2 \times 3 = 6$

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{6} = 30^\circ$$

Coil Span = $5 \times \beta = 150^\circ$

α = Short pitching angle $= 180^\circ - 150^\circ$

$= 30^\circ$

$$k_c = \cos\frac{\alpha}{2} = \cos\frac{30^\circ}{2} = \cos 15^\circ = 0.9659$$

$$kd = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{2 \times 30^\circ}{2}\right)}{2\sin\left(\frac{30^\circ}{2}\right)} = 0.9659$$

$$\boxed{kd = 0.9659}$$

(14) A 3-phase, 50 Hz, Star-Connected alternator with 2-layer winding is running at 600 rpm. It has 12 turns / coil, 4 Slots / pole / phase and a Coil-pitch of 10 Slots, if the flux / pole is 0.035 wb Sinusoidally distributed, find the phase and line emf's induced. Assume that the total turns / phase are series Connected.

Sol

$f = 50\ Hz$, $N_s = 600\ rpm$ 12 turns / coil

$\phi = 0.035\ Wb$ $m =$ Slots / pole / ph $= 4$

Coil pitch $= 10$ Slots

$n =$ Slots / pole $= m \times 3 = 4 \times 3 = 12$

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{12} = 15^\circ$$

$$\alpha = 2 \times (\text{Slot angle}) = 2\beta = 30^\circ$$

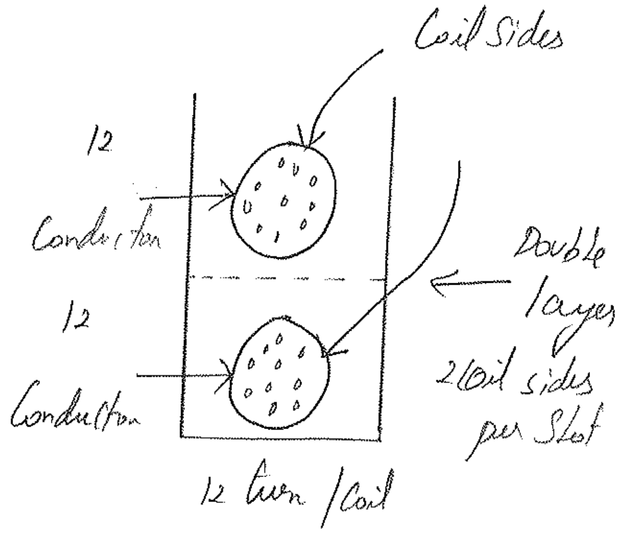
$$K_c = \cos\frac{\alpha}{2} = 0.9659$$

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\left(\sin\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{4 \times 15}{2}\right)}{4\sin\left(\frac{15}{2}\right)} = 0.9576$$

$$N_s = \frac{120f}{P} \Rightarrow 600 = \frac{120 \times 50}{P} = \boxed{P = 10}$$

number of Slots $= n \times p$

$= 12 \times 10 = 120$



Total Conductors / Slot $= 24$

$Z = 24 \times 120$

$$\boxed{Z = 2880}$$

$$\text{Conductors / phase} = \frac{2880}{3} = 960$$

$$T_{ph} = \frac{Z_{ph}}{2} = \frac{960}{2} = 480$$

$$E_{ph} = 4.44 K_c K_d \phi f T_{ph}$$

$$= 4.44 \times 0.9659 \times 0.9576 \times 0.035 \times 480 \times 50$$

$$E_{ph} = 3449.678 \text{ V}$$

$$E_{Line} = \sqrt{3} E_{ph}$$

$$= \sqrt{3} \times 3449.678$$

$$\boxed{E_{Line} = 5975.017 \text{ V}}$$

(15) Find the number of series turns required for each phase of a 3-phase, 50 Hz, 10-pole alternator with 90 slots. The winding is to be star-connected to give a line voltage of 11 kV. The flux/pole is 0.16 Wb

Sol :

$$f = 50\,Hz,\ p = 10,\ Slots = 90,\ E_{Line} = 11\,kV,\ \phi = 0.16\,Wb$$

$$E_{ph} = \frac{E_{Line}}{\sqrt{3}} = \frac{11}{\sqrt{3}} = 6.3508\,kV$$

$$n = \frac{Slots}{pole} = \frac{90}{10} = 9$$

$$m = Slots/pole/phase = 9/3 = 3$$

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ$$

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\left(\frac{3\times 20^\circ}{2}\right)}{3\sin\left(\frac{20^\circ}{2}\right)} = 0.9598$$

$$\boxed{K_c = 1}$$

$$E_{ph} = 4.44\,K_c\,K_d\,\phi\,f\,T_{ph}$$

$$6.3508\times 10^3 = 4.44\times 1\times 0.9598\times 0.16\times 50\times T_{ph}$$

$$T_{ph} = \frac{6.3508\times 10^3}{4.44\times 1\times 0.9598\times 0.16\times 50} = 186.28$$

$$T_{ph} \approx 186.$$